CHAPTER 11: RABBIT-COYOTE BIOLOGICAL MODEL COMPARISON

Let's now compare a continuous-time system model to its rule-based counterpart. We will use the classical example of biological balance between a host and a parasite as provided in many texts, e.g., Gordon, pp. 103†. In this example, the dynamics of the interactivity between the rabbit and coyote populations are modeled. In this model, rabbits are the *hosts* prey, multiplying in large numbers compared to coyotes who are the *parasites* hunters. The equations, when simplified, take the form described by Gordon as follows:

$$\frac{dr}{dt} = A \cdot r(t) - B \cdot r(t) \cdot c(t)$$

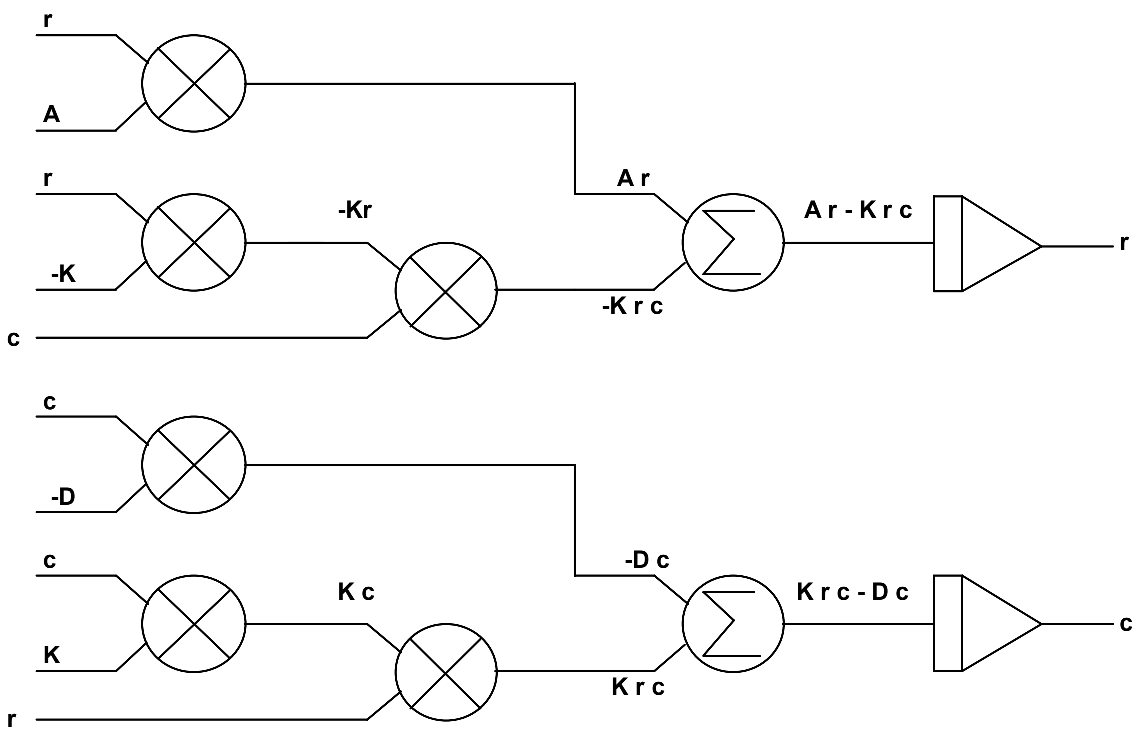
$$\frac{dc}{dt} = K \cdot r(t) \cdot c(t) - D \cdot c(t)$$

The first equation defines the rate of change of the rabbit population, where rabbit births are a fraction, A, of the existing population, and rabbit deaths (due to coyote kills) are a fraction, B, of the product of the rabbit and coyote populations. The coyote population changes similarly, but they are modeled as the birth rate being a fraction, K, of the product of the rabbit and coyote populations, and their death rate is a fraction, D, of their population.

The classical approach to determining these coefficients for this problem is to assume the solution to be quasi-stable, i.e., oscillatory, with no damping to a stable state. This is justified on the grounds that oscillation is observed in real life. However, as we shall show, this is not a realistic representation of a physical system, since any perturbation will drive the system into an unstable state, causing at least one of the populations to got to infinity or zero. In fact, basically stable systems can be considered to operate in constant oscillation, even though they require continuous perturbation from an external source. One merely has to redefine the external source as part of the overall system. Any form of clock or electronic oscillator is good example. This is fine when using simple mathematical models of oscillators as examples in a classroom environment, where the complexity of nonlinear models need not be described. However, it presents a misleading picture when trying to explain the real biological behavior of interest here.

† Geoffrey Gorgon, **System Simulation**, 2nd Ed., Prentice-Hall, Englewood Cliffs, NJ, 1978.

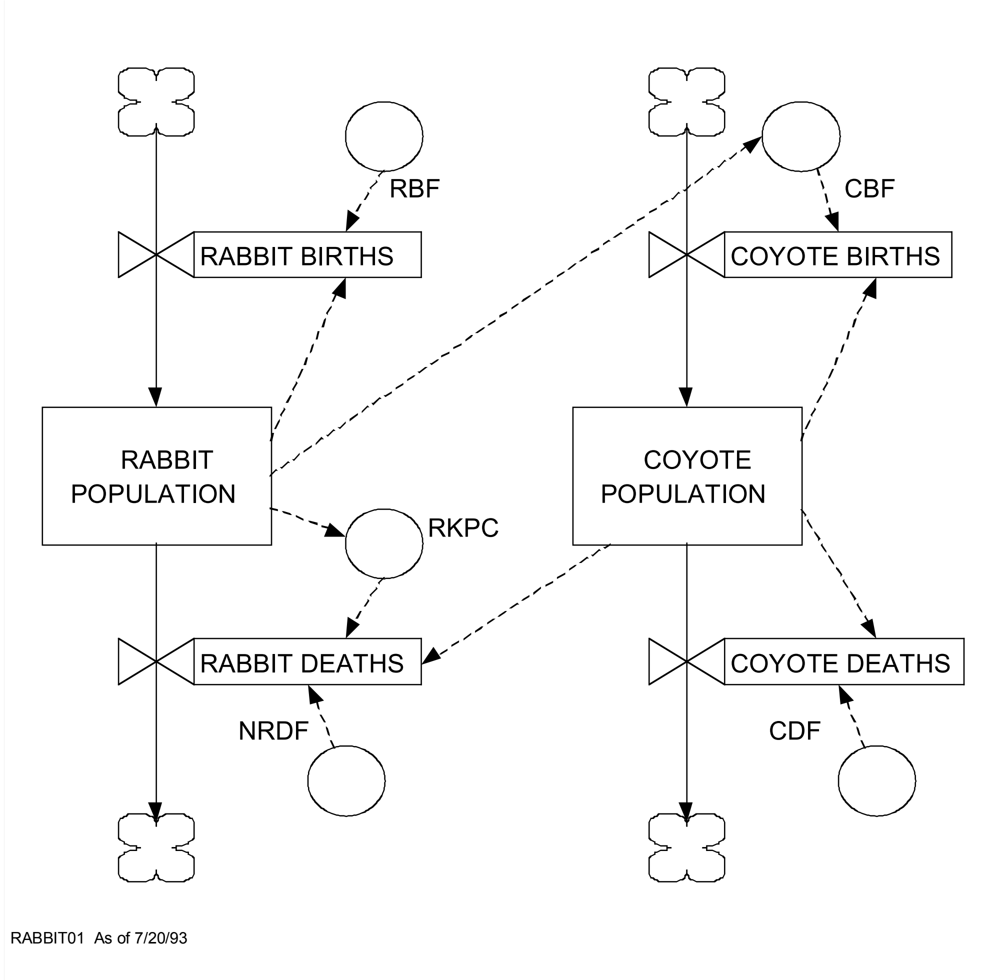
RABBIT - COYOTE BIOLOGICAL MODEL



RABBIT02 As of 2/23/93

This picture illustrates an approach to describing the model graphically using a fairly standard *analog diagram* for the differential equations. This set of equations can be solved using special methods or existing software systems. The analog diagram is easily related to the equations.

RABBIT - COYOTE BIOLOGICAL MODEL



This picture shows a *stock and flow diagram* for the same system, using slightly different coefficients for the equations. This diagram is somewhat more easily related to the stock and flow of rabbits and coyotes, but is harder to relate to the system of equations.

A MORE REALISTIC MODEL OF THE PHYSICAL PHENOMENON

The theory and design of electronic oscillators has been well researched. Their operational characteristics are governed by nonlinear physical phenomenon, refer to Hafner†. Accurate representation of real physical oscillatory behavior requires nonlinear models. In addition, damping exists in physical systems to some degree, as do external perturbations. When the perturbations are absent, the system will relax, with decreasing oscillatory behavior, to a stable state - normally not oscillatory because of the effects of damping. When a perturbation occurs, then the system moves from its stable state. These perturbations can came close enough together to cause superposition of their effects, and give the appearance of continual oscillatory motion. We submit that is the case with the typical biological model.

The model of a system that is less than critically damped will show the same form of oscillatory responses every time it is perturbed. Clearly, biological systems such as rabbits and coyotes are always being perturbed by external factors not modeled here. These can produce what would appear to be continuous oscillation, even though the systems themselves are highly stable. These additional perturbations would hardly change the overall behavior of the system. Depending on how one chooses the coefficients in the nonlinear equations, vastly different results can occur. One must study the effects of perturbations on the populations to gain good agreement with reality.

Given these facts, both of the linear models described above can misrepresent the real physical behavior of the biological system, particularly if one is concerned about studying the survival of the populations. It is more realistic to represent coyote deaths as due to starvation (not enough rabbit kills) and natural causes. Similarly, given reasonable circumstances, the rabbit population in a linear model quickly grows to infinity - an impossible consequence if we are studying a finite geographical area, with its finite food supply.

Rabbits will also starve if they don't have enough food, and can die of natural causes as well. Also, the incubation periods of different animals can be significantly different, affecting the time constants for birth after pregnancy. As we add these more detailed representations to gain accuracy, the model will necessarily become more complex. But, instead of solving a set of equations for fictitious coefficients to land on the single point of oscillation, we are providing real characterizations, observed phenomena and watching the simulated results. Furthermore, both of the prior approaches require a knowledge of how to transform the description of a physical problem into differential equation format. Our rule-based approach dispenses with this requirement. The description appears in a natural language format, requiring only a knowledge of algebra.

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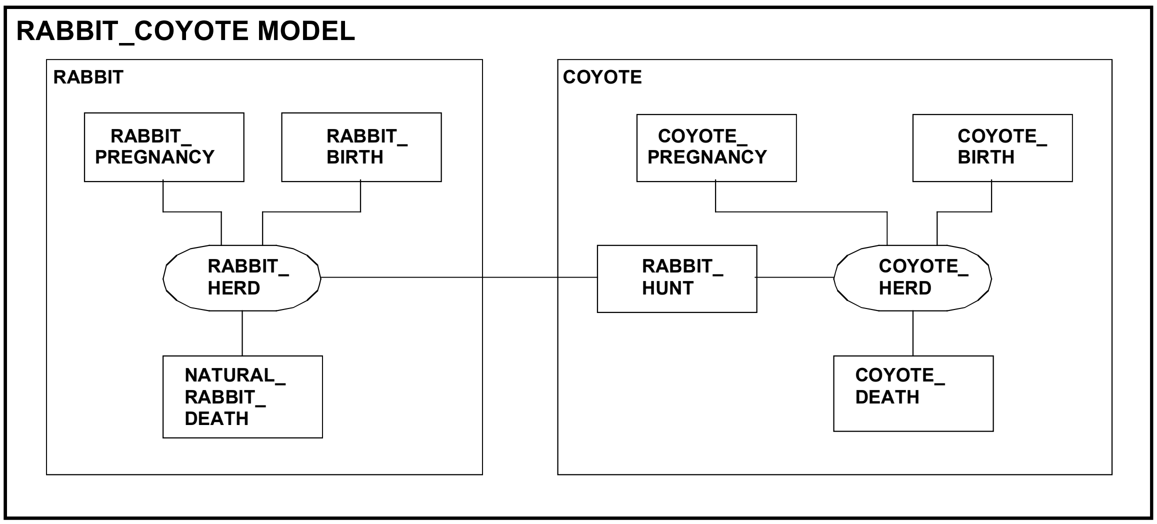
† Eric Hafner, *Theory and Design of Oscillators*, Proceedings of the IEEE, New York, NY, in two issues, 1976-78?

A RULE-BASED RABBIT - COYOTE BIOLOGICAL MODEL

The picture below shows a GSS Model drawing of the rabbit coyote biological mode. This approach is totally different than that using differential equations described in the prior section.

RABBIT - COYOTE BIOLOGICAL MODEL



RABBIT03 As of 11/8/93

The GSS Processes and Resources are shown on the next three pages. Although not many decision processes are represented in these simple models, the approach to the computations is more understandable in terms of real life considerations. One does not have to understand differential equations to represent the physical system. However, the models account for much more than their counterparts in the prior section. Equally important, the counterpart of nonlinear differential equations would be much more complex to write and solve.

Graphs of the dynamic behavior of the possible rabbit - coyote relationship, as represented in GSS, are shown in Figures 5.4 through 5.7. These results were obtained by modifying the birth, death, and hunger submodels for each. It can be seen that a wide range of results can be obtained, depending upon various factors. Figures 5.4 and 5.5 show the result of a single perturbation, at the beginning of the chart, on this very stable system. We note that a sequence of perturbations, *occurring two years apart*, will give the appearance of continuous oscillation. We would expect perturbations to occur at least this often.

*****Data for the Resource: RABBIT_HERD**

```
RABBIT
    1    POPULATION                  INTEGER INITIAL_VALUE 10000
    1    PREGNANCY_SET               INDEX   INITIAL_VALUE 1
    1    PREGNANCIES    QUANTITY(9)  INTEGER
    1    NATURAL_DEATHS              INTEGER
    1    TOTAL_DEATHS                INTEGER
    1    HUNGER_DEATHS               INTEGER
    1    HUNGER_DEATH_FACTOR         REAL
    1    DEATHS_BY_COYOTE            INTEGER
    1    REPRODUCTION_RATE           REAL    INITIAL_VALUE 0.2
    1    NATURAL_DEATH_RATE          REAL    INITIAL_VALUE 0.05
    1    MEDIAN_POPULATION           INTEGER INITIAL_VALUE 10000
```

*****Data for the Process: RABBIT_PREGNANCY**

```
PREGNANCY_CONTROL
  ***     DETERMINE PREGNANCY GROUP (MONTH - INSTANCE)
    IF PREGNANCY_SET IS GREATER THAN 2
        PREGNANCY_SET = 1.
  ***     COMPUTE NUMBER OF PREGNANCIES FOR THIS PEROID
    PREGNANIES (PREGNANCY_SET) = REPRODUCTION_RATE * POPULATION

  ***     SCHEDULE BIRTH AND PREGNANCY DATES
    SCHEDULE RABBIT_BIRTH IN 60 DAYS USING PREGNANCY_SET
    SCHEDULE RABBIT_PREGNANCY IN 30 DAYS USING PREGNANCY_SET
    INCREMENT PREGNANCY_SET.
```

*****Data for the Process: RABBIT_BIRTH**

```
RABBIT_BIRTH_CONTROL
    ADD PREGNANCIES(PREGNANCY_SET) TO POPULATION
```

*****Data for the Process: NATURAL_RABBIT_DEATH**

```
RABBIT_DEATH_CONTROL
    IF POPULATION IS GREATER THAN ZERO
        HUNGER_DEATH_FACTOR = ((MEDIAN_POPULATION + POPULATION)/
                                       MEDIAN_POPULATION)**2
        TOTAL_DEATHS = NATURAL_DEATH_RATE * POPULATION *
                                       HUNGER_DEATH_FACTOR
        SUBTRACT TOTAL_DEATHS FROM POPULATION.
    IF POPULATION IS LESS THAN 2
        STOP.
    SCHEDULE NATUAL_RABBIT_DEATH IN 30 DAYS
```

*****Data for the Resource: COYOTE_HERD**

```
COYOTE
    1   POPULATION                 INTEGER INITIAL_VALUE 250
    1   PREGNANCY_SET              INDEX   INITIAL_VALUE   1
    1   PREGNANCIES  QUANTITY(9)   INTEGER *** ALLOW UP TO 9 INSTANCES
    1   NATURAL_DEATHS             INTEGER
    1   HUNGER_DEATHS              INTEGER
    1   TOTAL_DEATHS               INTEGER
    1   RABBIT_KILLS               INTEGER
    1   HUNGER                     REAL
    1   COYOTE_RABBIT_RATIO        REAL
    1   REPRODUCTION_RATE          REAL    INITIAL_VALUE 0.1
    1   NATURAL_DEATH_RATE         REAL    INITIAL_VALUE 0.02
    1   HUNGER_DEATH_RATE          REAL    INITIAL_VALUE 0.02
    1   APPETITE                   INTEGER INITIAL_VALUE 20
    1   PROBABILITY_OF_CATCH       REAL
    1   MEDIAN_RABBIT_CATCH        INTEGER INITIAL_VALUE 10000

TIME_FACTORS
    1   DAY_COUNT                  INTEGER
    1   MONTH                      INTEGER
```

*****Data for the Process: COYOTE_PREGNANCY**

```
PREGNANCY_CONTROL

  ***  DETERMINE_PREGNANCY SET (MONTH - INSTANCES)
    IF PREGNANCY_SET IS GREATER THAN 3
        PREGNANCY_SET = 1.

   ***  SET COYOTE PREGNANCIES AND SCHEDULE BIRTH
    PREGNANCIES(PREGNANCY_SET) = REPRODUCTION_RATE * POPULATION
    SCHEDULE COYOTE_BIRTH IN 90 DAYS USING PREGNANCY_SET
    SCHEDULE COYOTE_PREGNANCY IN 30 DAYS USING PREGNANCY_SET
    INCREMENT PREGNANCY_SET.
```

*****Data for the Process: COYOTE_BIRTH**

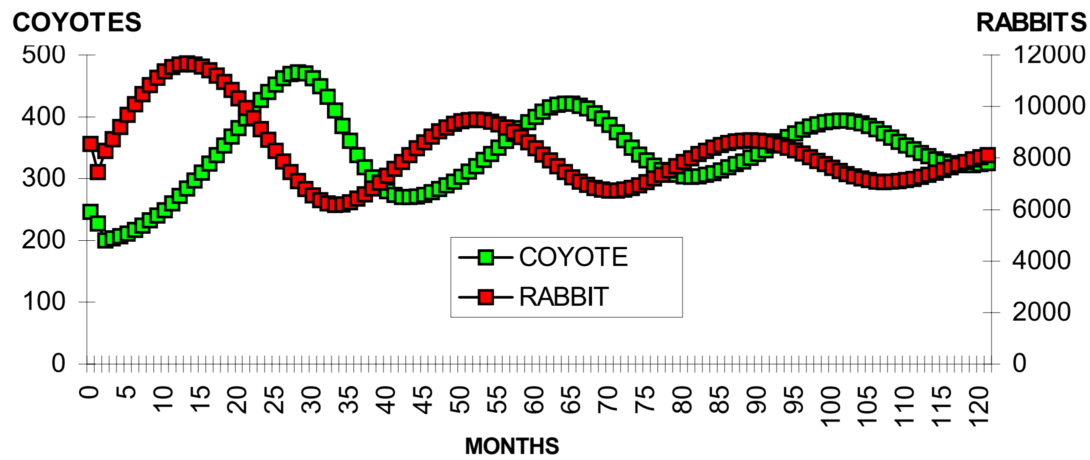
```
COYOTE_BIRTH_CONTROL
    ADD PREGNANCIES(PREGNANCY_SET) TO POPULATION
```

*****Data for the Process: COYOTE_DEATH**

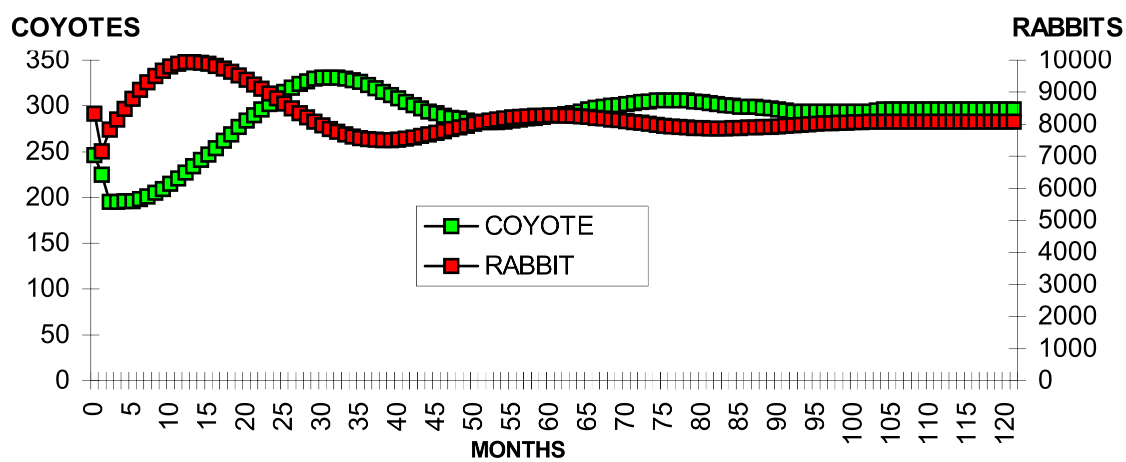
```
COYOTE_DEATH_CONTROL
    IF POPULATION IS GREATER THAN ZERO EXECUTE
        COMPUTE_COYOTE_DEATHS.
    IF POPULATION IS LESS THAN 2
        STOP.
    SCHEDULE COYOTE_DEATH IN 30 DAYS

COMPUTE_COYOTE_DEATHS
  ***     DETERMINE COYOTE HUNGER
    COYOTE HUNGER = 3*(5*COYOTE POPULATION/RABBIT_KILLS) ** 3
    NATURAL_DEATHS = NATURAL_DEATH_RATE * POPULATION
    HUNGER_DEATHS = COYOTE HUNGER * HUNGER_DEATH_RATE * POPULATION
    TOTAL_DEATHS = NATURAL_DEATHS + HUNGER_DEATHS
```

*****Data for the Process: RABBIT_HUNT**

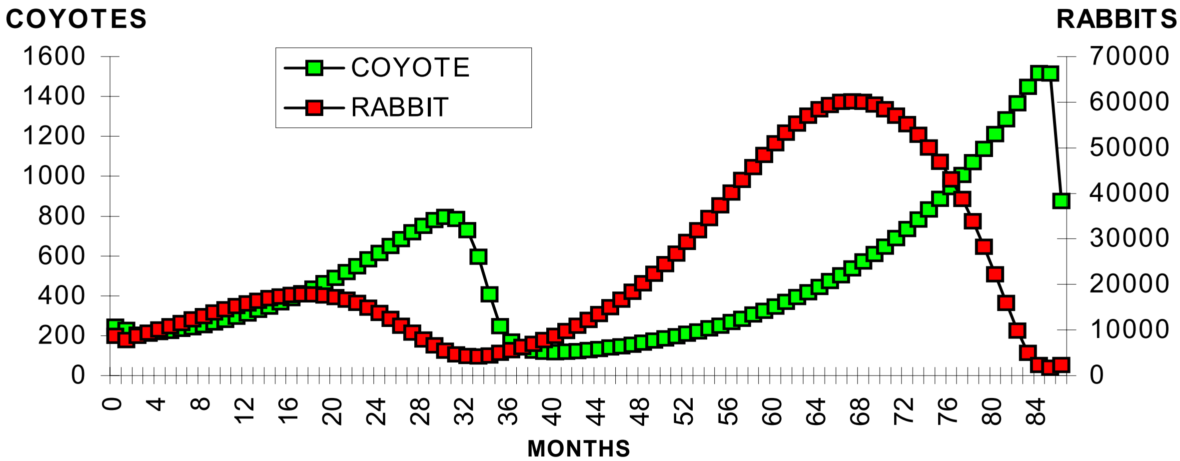
```
RABBIT_HUNT
  ***  DETERMINE RABBIT_KILLS
    PROBABILITY_OF_CATCH = (RABBIT POPULATION/
                        (RABBIT POPULATION + MEDIAN_RABBIT_CATCH)) ** 2
    RABBIT_KILLS = APPETITE * COYOTE POPULATION * PROBABILITY_OF_CATCH
    DECREMENT RABBIT POPULATION BY RABBIT_KILLS
    IF RABBIT POPULATION IS LESS THAN 2
        STOP.
    IF RABBIT_KILLS ARE LESS THAN ZERO
          RABBIT_KILLS = 1.
    SCHEDULE RABBIT_HUNT IN 30 DAYS
```



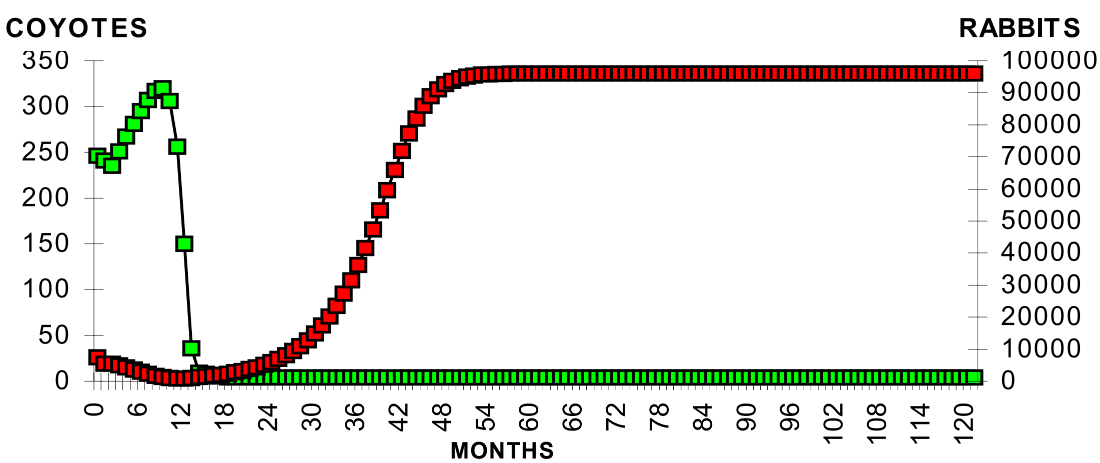
The graph above shows an oscillatory relationship when basic model is linear and rabbits are limited by food.



The graph above shows a stable relationship when basic model is nonlinear and rabbits are limited by food.



The graph above shows an unstable relationship when model is linear and rabbit growth is not limited.



The graph above shows that extinction occurs when model is linear and rabbit growth is slow and limited.