

Decibel Notation

The human ear is logarithmic in its response to sounds. By this, I mean that the apparent loudness of a sound is proportional to the *logarithm* of the sound's intensity. It is also interesting to note that the apparent pitch of a sound is proportional to the *logarithm* of its frequency; thus the equally-tempered chromatic musical scale divides one octave into twelve notes (counting both the whole- and half-steps) which are evenly spaced on a *log-frequency* scale. Alexander Bell was motivated by this to define the unitless quantity called the "Bel" as $P_{bel} = \log_{10} (P/P_{REF})$. In this usage, the acoustic power P is compared against a reference level P_{REF} (which has been previously agreed upon) to give the apparent loudness in "Bels." Note that a Bel has no units. It was quickly found that a Bel was too large, and the decibel (dB) was then defined as:

$$P_{dB} \equiv 10 \log_{10} \left| \frac{P}{P_{REF}} \right|$$

The magnitude signs are added to avoid taking the logarithm of a negative number. Decibel notation removes any indication of whether the quantity P is positive or negative.

Decibel notation is widely used in electronics because of the extremely wide range of power levels that might occur in the same system. As an example, consider a radio system in which the power levels might range from 1 kW at the transmitting antenna, to 10×10^{-12} W at the receiving antenna, to 1 W at the receiving loudspeaker. If we agree on a reference level of $P_{REF} = 1$ W, these numbers in decibel notation become +30 decibels, -110 decibels, and 0 decibels, respectively. In abbreviated form, these power levels become +30 dB, -110 dB, and 0 dB. When a 1-W reference is used, this is often indicated by writing "dBW." A 1-mW reference power may be indicated by "dBm."

Voltages and currents in an electronic signal processing system also tend to have a wide range of values. Because power is proportional to voltage or current squared, decibel notation for voltage or current is defined as follows:

$$V_{dB} = 10 \log_{10} \left| \frac{V}{V_{REF}} \right|^2 = 20 \log \left| \frac{V}{V_{REF}} \right| \quad \text{or} \quad I_{dB} = 20 \log_{10} \left| \frac{I}{I_{REF}} \right|$$

The input-output relationship of a signal processing system is known as its "transfer function." For an amplifier, the transfer function (that is, the output voltage or current expressed as a function of the input voltage or current) is simply called the gain. Gains or transfer functions are often expressed in dB notation, especially when they are very large or small. In this usage, the gain is given as:

$$A_{v,dB} = 20 \log_{10} \left| \frac{V_{out}}{V_{in}} \right| \quad \text{or} \quad A_{i,dB} = 20 \log_{10} \left| \frac{I_{out}}{I_{in}} \right|$$

As before, the sign of the gain function is lost when it is converted to dB notation.

Sample Voltage Gains and Their Decibel Equivalents

<u>Voltage Gain</u>	<u>dB Equivalent</u>		<u>Voltage Gain</u>	<u>dB Equivalent</u>
1	0 dB		2	+6 dB
10	+20 dB		4	+12 dB
1,000	+60 dB		1/2	-6 dB
1,000,000	+120 dB		1/4	-12 dB
-10	+20 dB		$\sqrt{2}$	+3 dB
0.1	-20 dB		$\frac{1}{\sqrt{2}}$	-3 dB
$\frac{1}{1,000,000}$	-120 dB			

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